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A note on non-Boussinesq plumes in an incompressible stratified environment

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The recent work of Rooney & Linden (1996) is generalized to describe the motion of non-Boussinesq plumes in both uniform and stratified environments. Using an integral model in which the horizontal entrainment velocity is assumed to take the form $u_\epsilon = \alpha(\overline{\rho}/\rho_e)^{1/2}w$ where α is the entrainment coefficient, $\overline{\rho}$ is the plume density, w the plume velocity and ρ_e the ambient density, it is shown that the vertical scale over which non-Boussinesq effects are significant is given by $z_B = \frac{5}{3}\left(B_o^2/(20\alpha^4 g^3)\right)^{1/5}$ where B_o is the buoyancy flux at the source. In a uniform environment, the system admits similarity solutions such that the location of the source of a real plume lies a distance $z_B|\rho_o/\Delta\rho|^{-5/3}$ beyond the point source of the similarity solution. The above entrainment law implies a fundamental difference between the motion of upward and downward propagating non-Boussinesq plumes, with the radius of upward propagating plumes being greater than that of the equivalent Boussinesq plume, while the radius of downward propagating plumes is smaller. In a stratified but incompressible environment the model predicts that non-Boussinesq effects are confined close to the source and that at each height, the plume velocity and the fluxes of mass, momentum and buoyancy coincide exactly with those of the equivalent Boussinesq plume. Furthermore, at the neutral buoyancy height, the plume radius equals that of the equivalent Boussinesq plume.

1. Introduction

Experimental data of Ricou & Spalding (1961) and Thring & Newby (1953) suggest that in an ascending non-Boussinesq plume, entrainment of ambient fluid is influenced by the difference in density between the plume and the environment as well as the plume velocity. The horizontal entrainment velocity has the form

$$u_e = \alpha(\overline{\rho}/\rho_e)^{1/2}w \tag{1}$$

where α is the entrainment coefficient, $\overline{\rho}$ is the plume density, w the plume velocity and ρ_e the ambient density. Rooney & Linden (1996) recently showed that, if the entrainment has this form, then the motion of a non-Boussinesq plume of gas rising through a uniform incompressible environment from a point source of buoyancy is self-similar. This self-similarity provides a powerful means of describing the non-Boussinesq effects which arise in hot fire plumes, buoyant volcanic eruption columns and hydrothermal plumes on the sea-floor. However, in a number of these situations, the ambient density gradient plays an important role in confining the ascent of the plume. It is therefore of interest to examine the interplay between non-Boussinesq effects and ambient stratification.

To this end, we first describe the self-similar motion of non-Boussinesq plumes, calculating the length scale over which non-Boussinesq effects are important and illustrating that such plumes asymptote to an equivalent Boussinesq plume over larger scales. We then study the importance of non-Boussinesq effects on the motion of plumes rising through a stratified environment. Finally, we apply the model to a range of phenomena in which the source fluid may introduce large density contrasts.

2. The model of a non-Boussinesq plume

We consider a general fluid in which the density ρ varies with a dimensionless fluid property C according to a law of the form

$$\frac{1}{\rho} = \lambda + \beta C \tag{2}$$

where $\lambda, \beta > 0$. This law may be used to describe (i) the density of a particle-laden liquid or isothermal gas, where $1-C$ represents the mass fraction of particles, $1/(\lambda+\beta)$ represents the density of the host fluid and $1/\lambda$ represents the density of the particles; (ii) the density of a mixture of two linearly mixing fluids, where C is the mass fraction of one end-member fluid, $1/(\lambda+\beta)$ is the density of this fluid and $1/\lambda$ is the density of the second end-member fluid; (iii) the density of an ideal gas of temperature CT_o where $\lambda = 0$ and $\beta = RT_o/P$ where R is the gas constant and T_o is the initial plume temperature (this is the case considered by Rooney & Linden 1996).

We follow Morton, Taylor & Turner (1956) and adopt the top-hat model of a plume in which the average plume velocity, $\overline{w}(z)$, density, $\overline{\rho}(z)$, and concentration, $\overline{C}(z)$, and the effective plume radius, $b(z)$, are defined by the horizontal averages of velocity, $w(r,z)$, density $\rho(r,z)$ and concentration $C(r,z)$ according to

$$Q = \overline{\rho}\overline{w}b^2 = \int_0^\infty 2\rho w r \,\mathrm{d}r, \quad M = \overline{\rho}\overline{w}^2 b^2 = \int_0^\infty 2\rho w^2 r\,\mathrm{d}r, \tag{3a,b}$$

$$\overline{\rho}\overline{w}\overline{C}b^2 = \int_0^\infty 2C\rho w r\,\mathrm{d}r, \quad \overline{\rho} = \frac{1}{\lambda + \beta\overline{C}}. \tag{3c,d}$$

As the plume moves through an incompressible environment, in which $C = C_e$ and $\rho = \rho_e$ say, then, using the entrainment law (1), it follows that the conservation of mass flux, momentum flux and the flux of quantity C (for example: (i) particles, (ii) one end-member fluid or (iii) specific enthalpy) take the form (cf. Morton *et al.* 1956)

$$\frac{\mathrm{d}Q}{\mathrm{d}z} = 2\alpha\overline{w}b(\overline{\rho}\rho_e)^{1/2}, \tag{4}$$

$$\frac{\mathrm{d}M}{\mathrm{d}z} = g(\rho_e - \overline{\rho})b^2 = g\beta(\overline{C} - C_e)b^2\rho_e\overline{\rho}, \tag{5}$$

$$\frac{\mathrm{d}\overline{C}Q}{\mathrm{d}z} = 2\alpha\overline{w}b(\overline{\rho}\rho_e)^{1/2}C_e, \tag{6}$$

where for case (iii) the specific heat at constant pressure is assumed constant.

For an upward propagating plume, we define the local buoyancy flux

$$B = g\frac{\rho_e - \overline{\rho}}{\rho_e}\overline{u}b^2 = g\beta(\overline{C} - C_e)Q. \tag{7}$$

Combining equations (4), (6) and (7), it follows that

$$\frac{\mathrm{d}B}{\mathrm{d}z} = -g\beta \frac{\mathrm{d}C_e}{\mathrm{d}z} Q, \tag{8}$$

where we assume that β is constant (equation (2)). Equations (4) and (5) may also be re-expressed in the more compact form

$$\frac{\mathrm{d}Q}{\mathrm{d}z} = 2\alpha M^{1/2} \rho_e^{1/2}, \tag{9}$$

$$\frac{\mathrm{d}M}{\mathrm{d}z} = \frac{BQ\rho_e}{M}. \tag{10}$$

Equations (8)–(10) model the motion of a non-Boussinesq plume, and are analogous to the model proposed by Rooney & Linden (1996). In the model, the plume fluid may undergo large changes in density, and therefore it has application in many environmental situations, for example plumes produced by gas leaks or small fires.

3. Similarity solutions in uniform environments

3.1. *Upward propagating non-Boussinesq plumes in an unstratified environment*

In the limit of no ambient stratification, $\mathrm{d}C_e/\mathrm{d}z = 0$ and the buoyancy flux B is a constant. Now equations (9), (10) admit similarity solutions (Morton *et al.* 1956; Rooney and Linden 1996)

$$Q = \frac{6\alpha}{5} \left(\frac{9\alpha}{10} \right)^{1/3} \rho_e B^{1/3} z^{5/3}, \tag{11}$$

$$M = \left(\frac{9\alpha}{10} \right)^{2/3} \rho_e B^{2/3} z^{4/3}. \tag{12}$$

Combining these solutions with the density relation (3*d*) we obtain explicit expressions for b, $\overline{w}$, $\overline{\rho}$ and $\overline{C}$:

$$b = \frac{6\alpha}{5} z \left(1 + \left(\frac{z_B}{z} \right)^{5/3} \right)^{1/2}, \quad \overline{\rho} = \frac{\rho_e}{1 + \left(z_B/z \right)^{5/3}} \tag{13a,b}$$

$$\overline{w} = \left(\frac{9\alpha}{10} \right)^{1/3} \left(\frac{5}{6\alpha} \right) B^{1/3} z^{-1/3}, \quad \overline{C} = C_e + \frac{1}{\rho_e \beta} \left(\frac{z_B}{z} \right)^{5/3}, \tag{13c,d}$$

where the length scale

$$z_B = \frac{5}{3} \left(\frac{B^2}{20\alpha^4 g^3} \right)^{1/5}. \tag{14}$$

By analogy with Boussinesq plumes, the entrainment coefficient $\alpha \sim 0.1$.

The length scale z_B represents the length scale over which non-Boussineq effects are important. Over this length scale, the plume adjusts to Boussinesq behaviour through entrainment, and so for plumes of larger buoyancy flux, a greater mass of ambient fluid needs to be entrained for the density to converge to that of the environment, tending to increase z_B. The length scale, z_B, also depends on the gravitational acceleration g since this controls the initial plume acceleration, and hence the plume speed and rate of entrainment. For $z \gg z_B$, the solution (13) asymptotes to the equivalent Boussinesq plume of the same buoyancy flux B, which issues from a point source

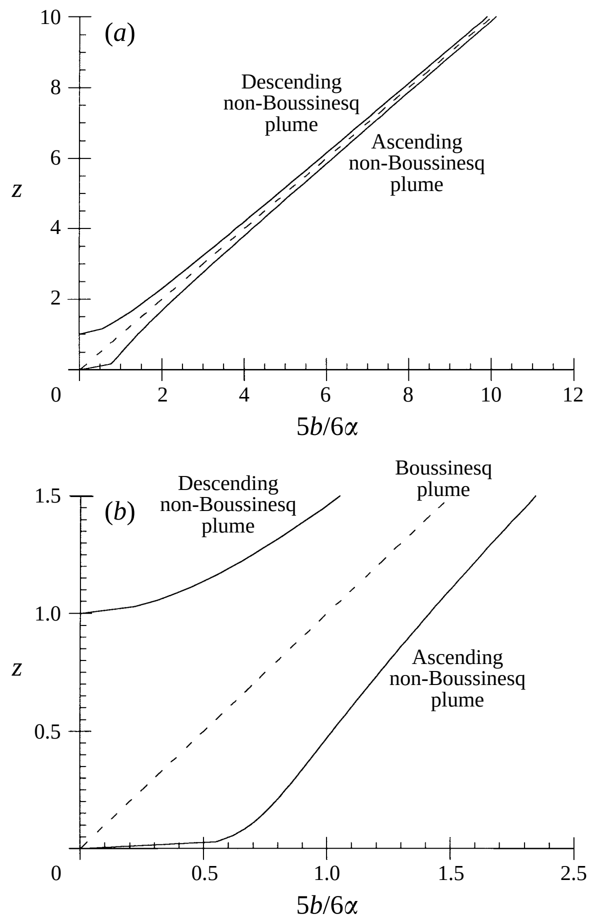

FIGURE 1. Variation of the radius (plotted as $5b/6\alpha$, cf. 13*a*), with position for both an ascending and descending non-Boussinesq plume: (*a*) the far-field structure which converges to the Boussinesq plume solution and (*b*) the structure near the source. Here $z_B = 1$. The descending plume solution only applies in the region $z > z_B = 1$.

located at $z = 0$ (figure 1; cf. Morton *et al.* 1956). The limit $z_B = 0$ corresponds to the classical solutions of Morton *et al.* (1956), which coincide with the equivalent Boussinesq plume.

The solution (equation (13); figure 1*a*) identifies that just above the source, where $z \ll z_B$, the plume expands very rapidly in comparison to the equivalent Boussinesq plume whose radius increases steadily with height at a rate $6\alpha/5$. Essentially, near the source, the density of the plume is very small relative to the environment (figure 2), and so since the mass flux of the plume is the same as the equivalent Boussinesq plume ((12); cf. Morton *et al.* 1956), the volume flux is greater. In turn, this causes the radius to increase more rapidly with height that in the equivalent Boussinesq plume, $\mathrm{d}b/\mathrm{d}z > 6\alpha/5$. However, higher in the plume, as entrainment causes the plume density to increase towards that in the environment (figure 2), the radius of the plume begins to evolve back towards that of the equivalent Boussinesq plume, since the total mass flux of the plume must coincide with that of the equivalent Boussinesq

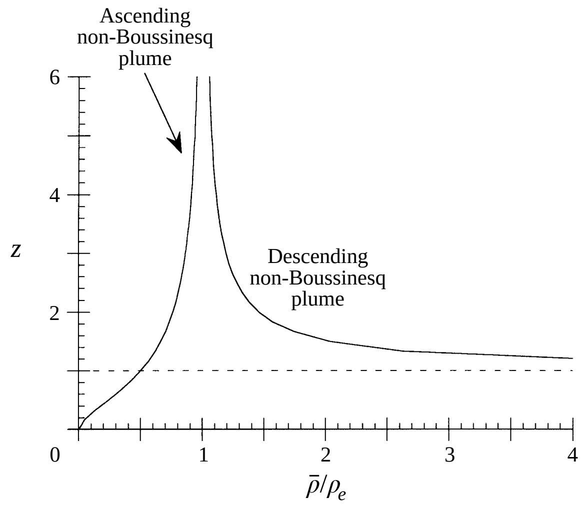

FIGURE 2. Variation of the dimensionless density of the plume, $\overline{\rho}/\rho_e$, with position for both a descending and an ascending non-Boussinesq plume. $z_B = 1$.

plume (equation (12)). Indeed, above the point $z = z_B/(24)^{3/5}$, the radius increases less rapidly with height than in the equivalent Boussinesq plume, $\mathrm{d}b/\mathrm{d}z < 6\alpha/5$.

The formal limit of the similarity solution (13) as $z \to 0$ gives

$$b \sim \frac{6\alpha}{5} z_B^{5/6} z^{1/6} \to 0, \quad \overline{\rho} \sim \rho_e \left(\frac{z}{z_B}\right)^{5/3} \to 0, \quad \overline{C} - C_e = \frac{1}{\beta\rho_e}\left(\frac{z_B}{z}\right)^{5/3} \to \infty. \tag{15}$$

However, in a real experiment, $\overline{C}$ is bounded at the physical source, $\overline{C} = C_o = 1/\beta\left(1/\overline{\rho}_o - \lambda\right)$ (equation (3*d*)) where $\overline{\rho}_o$ is the density at the source. Therefore, according to the similarity solutions, the real experimental source is located at $z = z_s$, a distance z_s above the (conceptual) point source where

$$z_s = z_B \left(\frac{\overline{\rho}_o}{\rho_e - \overline{\rho}_o}\right)^{3/5}; \tag{16}$$

z_s/z_B is shown as a function of $\overline{\rho}_o/\rho_e$ in figure 3. Even though the mathematical solutions start at $z = 0$, the solutions are only physically relevant in the region $z \geqslant z_s$ which corresponds to the actual plume – the mathematical solution in the region $0 < z < z_s$ simply determines the source conditions at $z = z_s$ such that the motion is self-similar in the region $z \geqslant z_s$. The modified entrainment law (1) therefore need only apply in the region $z > z_s$, where it is supported by experimental data (e.g. Thring & Newby 1953).

3.2. *Downward propagating non-Boussinesq plumes*

The entrainment law (1) implies that for upward propagating plumes, the entrainment is suppressed owing to the relatively small density of the plume fluid, whereas for downward propagating, relatively dense plumes, (1) suggests that the entrainment is enhanced. We therefore expect rather different behaviour for a downward propagating relatively dense plume. In this case, it is convenient to redefine the local buoyancy flux (cf. (7)) to be $B = g\beta(C_e - \overline{C})Q > 0$. If z is now measured downwards, then equations (8)–(10) again govern the motion, which may be described by further

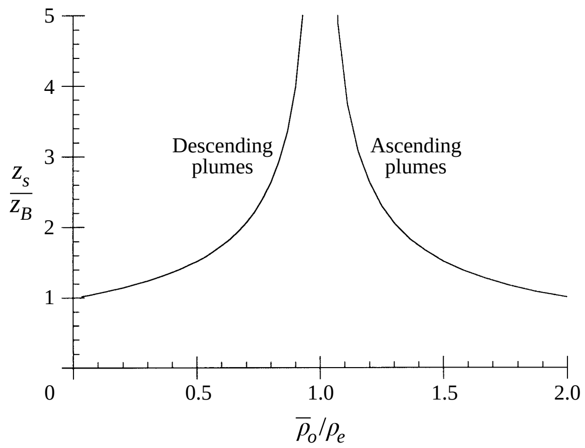

FIGURE 3. Location of the real source, z_s/z_B, as a function of the density ratio $\overline{\rho}_o/\rho_e$ at the source. Curves are shown for both upward and downward propagating sources. In real situations, $\overline{\rho}_o/\rho_e$ may take values in the range 0.5 (fire) to 1.2–1.3 (dense gas plume).

similarity solutions in which the radius, density and concentration are given by

$$b = \frac{6\alpha z}{5}\left(1 - \left(\frac{z_B}{z}\right)^{5/3}\right)^{1/2}, \quad \overline{\rho} = \frac{\rho_e}{1 - \left(z_B/z\right)^{5/3}}, \quad C_e - \overline{C} = (\rho_e\beta)^{-1}\left(\frac{z_B}{z}\right)^{5/3}. \quad (17)$$

As may be seen in figures 1 and 2, the model solution for the downward propagating plume is quite distinct from that for an upward propagating non-Boussinesq plume. The presence of the negative sign in equations (17) represents the fact that the density in the downward propagating plume exceeds that in the environment (figure 2). The solutions are only physically meaningful for $z > z_B$ since at the point $z = z_B$, the radius of the plume $b = 0$ and the plume density $\overline{\rho} \to \infty$ (figures 1 and 2). Note that at $z = z_B$, the mass and momentum fluxes are finite. Indeed, if the concentration of a real experimental plume at the source $C_o = (1/\beta)(1/\overline{\rho}_o - \lambda)$ then the plume is described by the similarity solution (17) in the region $z > z_s$ where

$$z_s = z_B\left(\frac{\overline{\rho}_o}{\overline{\rho}_o - \rho_e}\right)^{3/5} > z_B. \quad (18)$$

Figure 3 also shows the values z_s/z_B as a function of $\overline{\rho}_o/\rho_e$ in a descending plume. Note again that in order for these similarity solutions to model a real plume issuing from a source located at $z = z_s$, the modified entrainment law (equation (1)) need only apply in the region $z \geqslant z_s$. The mathematical solution in the region $z < z_s$ simply prescribes the source conditions at $z = z_s$ in order that the flow is self-similar for $z > z_s$.

For $z \gg z_B$ the non-Boussinesq effects become negligible and the plume asymptotes to a Boussinesq plume of buoyancy flux B_o, whose virtual point source is located at $z = 0$ (figure 1*a*). In fact, in a non-Boussinesq descending plume, $\mathrm{d}b/\mathrm{d}z > 6\alpha/5$ if $z > z_B$, and the plume radius converges monotonically to that of the equivalent Boussinesq plume. Note that the radii of upward and downward propagating non-Boussinesq plumes evolve in a qualitatively similar fashion to plumes issuing from distributed and constricted sources (cf. Caulfield & Woods 1995).

4. Effects of uniform stratification

We restrict attention to motion over vertical scales z smaller than the scale height of the environment,

$$z \ll z_c = \frac{\rho_e}{\mathrm{d}\rho_e/dz} = -\frac{1}{\rho_e \beta \mathrm{d}C_e/\mathrm{d}z} \tag{19}$$

so that over the scale of the plume, the ambient fluid remains incompressible and of nearly constant density. Thus ρ_e may be taken as a constant in equations (9) and (10), and equation (8) may be rewritten in terms of the Brunt–Väisälä frequency

$$N^2 = g\rho_e\beta\frac{\mathrm{d}C_e}{\mathrm{d}z} \tag{20}$$

as

$$\frac{\mathrm{d}B}{\mathrm{d}z} = -\frac{N^2 Q}{\rho_e}. \tag{21}$$

It is convenient to work with dimensionless mass, momentum and buoyancy fluxes defined by

$$\hat{Q} = B_o^{-3/4} N^{5/4} \rho_e^{-1} Q, \quad \hat{M} = B_o^{-1} N \rho_e^{-1} M, \quad \hat{B} = B_o^{-1} B \tag{22a–c}$$

and the dimensionless height

$$y = B_o^{-1/4} N^{3/4} z \tag{23}$$

where B_o is the buoyancy flux evaluated at the source. Equations (8)–(10) then become

$$\frac{\mathrm{d}\hat{Q}}{\mathrm{d}y} = 2\alpha\hat{M}^{1/2}, \quad \frac{\mathrm{d}\hat{M}}{\mathrm{d}y} = \frac{\hat{B}\hat{Q}}{\hat{M}}, \quad \frac{\mathrm{d}\hat{B}}{\mathrm{d}y} = -\hat{Q}. \tag{24a–c}$$

These equations are equivalent to those employed by Morton *et al.* (1956) to describe the volume flux, momentum flux (per unit mass) and buoyancy flux for a Boussinesq plume. Therefore, at each height $\hat{Q}$, $\hat{M}$ and $\hat{B}$ may be identified with the analogous dimensionless properties in a Boussinesq plume issuing from $y = 0$ with buoyancy flux B_o. Thus the height of rise (at which $\hat{M} = 0$) and the speed at each height, M/Q, are identical to those in the equivalent Boussinesq plume.

However, by combining (8), (10) and (22) it can be shown that in the non-Boussinesq plume

$$\overline{\rho} = \frac{\rho_e}{1 \pm \mathscr{A}\left(\hat{B}/\hat{Q}\right)}, \quad b = b_B\left(1 \pm \mathscr{A}\frac{\hat{B}}{\hat{Q}}\right)^{1/2} \tag{25a,b}$$

with the $\pm$ corresponding to ascending and descending plumes, and where b_B corresponds to the radius of the equivalent Boussinesq plume (cf. Morton *et al.* 1956). The parameter

$$\mathscr{A} = \frac{B_o^{1/4} N^{5/4}}{g} = \frac{z_a}{z_s} \tag{26}$$

represents the ratio of the height of rise of the plume in a stratified environment, $z_a \sim B_o^{1/4} N^{-3/4}$ (cf. Morton *et al.* 1956), to the scale height of the environment, $z_s = g/N^2$. The assumption of incompressibility requires $z_a \ll z_s$ and so $\mathscr{A} \ll 1$. From (25), we deduce that non-Boussinesq effects are only important in a region just above the source and that the nature of the non-Boussinesq effects is similar

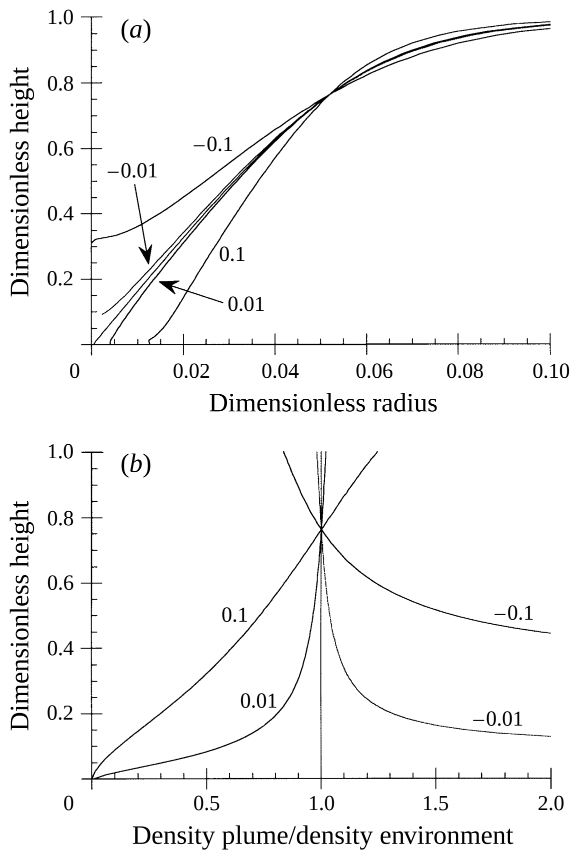

FIGURE 4. Variation of the (*a*) radius and (*b*) density with height in an ascending and a descending non-Boussinesq plume in a stratified incompressible environment. The heights and radii have been normalized with respect to the dimensionless height of the Boussinesq plume, for which $\mathscr{A} = 1$ and $y = 9.5$. In both cases curves are shown for $\mathscr{A} = 0.01$ and 0.1 with the descending plumes denoted by a minus sign. The ascending plume has a larger radius than the equivalent Boussinesq plume owing to the rapid non-Boussinesq expansion just above the source (cf. figure 1).

to that in a uniform environment (figure 1). Indeed, figure 4 illustrates that for an ascending plume there is an initial rapid expansion to radii in excess of the equivalent Boussinesq plume (curves $\mathscr{A} = 0.01, 0.1$), but that the motion then converges towards that of a Boussinesq plume. Essentially, the mass flux increases at the same rate as in the equivalent Boussinesq plume (24*a*), and so since near the source the non-Boussinesq plume has very small density, it has a much greater volume flux than the equivalent Boussinesq plume. In turn, this leads to a more rapid increase in radius with height (cf. figure 1*b*, §3). However, as the plume density increases towards that in the environment, the plume radius converges back towards that of the Boussinesq plume in order that the mass flux remains the same as in the equivalent Boussinesq plume; ultimately, at the neutral buoyancy height, $B = 0$, the radius coincides exactly with that of the equivalent Boussinesq plume (figure 4). Note that in the region above the neutral buoyancy height, the negative buoyancy decelerates the plume to rest.

In a descending non-Boussinesq plume, the model solutions only become physically relevant above some height $y_B(\mathscr{A}) > 0$ (cf. figure 1) at which the plume radius, $b = 0$ (figure 4, curves −0.01, −0.1). Since $\mathscr{A} \ll 1$, the non-Boussinesq effects occur close the source and are largely independent of the ambient stratification. Therefore

$$y_B \sim \frac{5}{3}\left(\frac{1}{20\alpha^4}\right)^{1/5} \mathscr{A}^{3/5} \quad \text{as} \quad \mathscr{A} \to 0. \tag{27}$$

Note that at the point $y = y_B$ the plume has finite mass, momentum and buoyancy flux even though $b = 0$. As the plume descends, it is initially denser than the environment, and so its radius is smaller than the equivalent Boussinesq plume since both plumes have the same mass flux. As the plume density converges towards that of the environment, the radius of the plume therefore increases towards that of the equivalent Boussinesq plume. Again, at the neutral buoyancy height, the plume radius exactly matches that of the equivalent Boussinesq plume.

5. Importance of non-Boussinesq effects

The adjustment of the radius to within about 10% of a Boussinesq plume occurs over the distance ((13*a*), (14)) $4z_B \sim 4B_o^{2/5}g^{-3/5}$ above the conceptual point source $z = 0$. Therefore if the real source is located at z_s, then the non-Boussinesq effects will only be significant in the region $z_s = z_B\left(\overline{\rho}_o/(\overline{\rho}_o - \rho_e)\right)^{3/5} < z < 4z_B$.

For a typical laboratory experiment using aqueous solutions, $\overline{\rho}_o - \rho_e \leqslant 0.1\rho_e$, and so $z_s/z_B \geqslant 4$–5. Therefore, even close to the source, non-Boussinesq effects are small. Further, since $B_o/\rho_e \leqslant 10^{-4}$–$10^{-8}$ m^5 s^{-3}, $z_B \leqslant 1$–2 cm. Hence even if larger initial density contrasts were possible, as in a bubble or particle-laden plume, the motion will adjust to Boussinesq behaviour within 5–10 cm of the source.

A small household or building fire may produce a heat flux Q_T of order 10^4 W and the fire temperature may exceed that of the surroundings by $\Delta T \sim 200$–300 K, so that $\overline{\rho}_o/\rho_e \sim 0.5$ and $z_s \sim z_B$. The smoke plume associated with this is therefore likely to have a region of very non-Boussinesq behaviour near the source. Indeed, for such a fire we may write $B_o = gQ_T/\overline{\rho}_o c_p T_o$ which has value $B_o \sim 1.0$ m^4 s^{-3} so that $z_B \sim 0.3$ m. Thus the plume behaves in a non-Boussinesq fashion in a region at most of order 1 m above the source. This may be a significant fraction of the depth of a room/building hosting the fire. However, we note that since the mass flux entrained into the plume, $\mathrm{d}Q/\mathrm{d}z$, varies exactly as that of the equivalent Boussinesq plume (equation (2)), the rate at which such a non-Boussinesq plume can restratify the environment through the filling box process (i.e. the speed of the first front; cf. Baines & Turner 1969) is identical to that of the equivalent Boussinesq plume. This is a consequence of the increased radius of the non-Boussinesq (ascending) plume which exactly compensates for the decreased turbulent entrainment rate.

Although in some geophysical situations, the density law may be more complex than (2), it is interesting to estimate the value of the effective source position z_s and the adjustment length z_B which may also be deduced from dimensional arguments. For example, in a hydrothermal plume, B_o may take values of order 0.1–1.0 m^4 s^{-3} and the hot hydrothermal fluid may issue into the ocean with a density $\overline{\rho}_o \geqslant 0.5\rho_e$. Therefore, $z_s \sim z_B$ and $z_B \sim 0.1$–1 m, so that the motion will converge to that of a Boussinesq plume over the first 1–5 m, which is a relatively small fraction of the total ascent height of 200–300 m (Turner & Campbell 1987). In a volcanic eruption column, just above the volcano, the plume density may decrease to values of order $\overline{\rho}_o \sim 0.2\rho_e$ so that $z_s \sim 0.5z_B$. The buoyancy flux associated with such plumes may then be as large as $B_o \sim 10^6$–10^9 m^4 s^{-3} and so $z_B \sim 100$–1000 m, suggesting that an eruption column remains non-Boussinesq over heights of order 1–5 km, consistent with the results of numerical models (Woods 1988).

6. Summary

Non-Boussinesq effects may have a great impact on the shape and density evolution of a plume over a distance z_B above the source. We estimate that this adjustment distance may be as large as 1 m in smoke plumes produced by small fires in buildings, and it may extend several kilometres in large volcanic eruption columns. However, the self-similar fluxes of mass, momentum and density deficit in a non-Boussinesq plume are identical to those of the equivalent Boussinesq plume. In a stratified incompressible environment, non-Boussinesq effects are confined to a region close to the source, and they do not influence the final ascent height of the plume. The solutions presented in this note are based on the entrainment law (1); it would be of interest to explore this parameterization with further experiments, examining the influence of initial momentum and mass flux on the entrainment process. In the different limit in which the plume height is greater than the scale height of the environment, $z_a \geqslant z_s$, $\mathscr{A} \geqslant 1$, non-Boussinesq effects may be important throughout the height of the plume. However, owing to the importance of compressibility, the underlying model becomes more complex since ρ_e varies over the height of the plume.

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